Course Code: 20HS0380 **R20**

SIDDHARTH GROUP OF INSTITUTIONS:: PUTTUR
(AUTONOMOUS)
Siddharth Nagar, Narayanavanam Road – 517583

QUESTION BANK (DESCRIPTIVE)

Subject with Code: Algebra and Calculus (20HS0830) **Course & Branch**: B.Tech - Common to all

Year & Sem: I-B.Tech & I-Sem **Regulation:** R20

UNIT –I
MATRICES

1	a) Reduce the matrix A= $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$ into Echelon form and find its rank?	[L3][CO1]	**[6M]**
	b) Show that the equations $x+y+z=4\,; 2x+5y-2z=3\,; x+7y-7z=5$ are not consistent.	[L2][CO1]	**[6M]**
2	a) Reduce the matrix A= $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ into Echelon form and find its rank?	[L3][CO1]	**[6M]**
	b) Find the Eigen value and Eigen vectors of the matrix $\begin{bmatrix} 5 & -2 & 0 \\ -2 & 6 & 2 \\ 0 & 2 & 7 \end{bmatrix}$	[L1][CO1]	**[6M]**
3	a) Define the rank of the Matrix.	[L1][CO1]	**[2M]**
	b) Find whether the following equations are consistent if so solve them $x+y+2z=4\,; 2x-y+3z=9\,; 3x-y-z=2.$	[L3][CO1]	**[10M]**
4	a) Reduce the matrix A= $\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ into Echelon form and find its rank?	[L2][CO1]	**[6M]**
	b) Determine the Eigen values of A^{-1} $where$ $A=\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$	[L5][CO1]	**[6M]**
5	Show that the only real number λ for which the system $x+2y+3z=\lambda x\,; 3x+y+2z=\lambda y\,; 2x+3y+z=\lambda z$ has non-zero solution is 6 and solve them when λ=6.	[L2][CO1]	**[12M]**
6	a) Solve completely the system of equations x+2y+3z=0, 3x+4y+4z=0, 7x+10y+12z=0.	[L3][CO1]	**[6M]**
	b) Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$	[L2][CO1]	**[6M]**
7	a) Find the Eigen values and corresponding Eigen vectors of the matrix $A=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.	[L1][CO1]	**[6M]**

	b) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$.	[L2][CO1]	**[6M]**
8	Find the Eigen values and corresponding Eigen vectors of the matrix A and also A^{-1} where $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.	[L1][CO1]	**[12M]**
9	a) State Cayley-Hamilton theorem.	[L1][CO1]	**[2M]**
	b) Show that the matrix $A = \begin{bmatrix} 1 & -2 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$ satisfies its characteristic equation and find A^{-1} ?	[L2][CO1]	**[10M]**
10	Verify Cayley Hamilton theorem for $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and find A^{-1} *and* A^4 using C-H theorem.	[L3][CO1]	**[12M]**

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UNIT –II
MULTI VARIABLE CALCULUS

1	a) Verify Rolle's Theorem for the function f(x) = $\frac{\sin x}{e^x}$ in $(0,\pi)$	[L2][CO2]	**[6M]**
	b) Verify Lagrange's mean value theorem for f(x) = $\log_e x$ in $[1,e]$.	[L2][CO2]	**[6M]**
2	a)State and verify Rolle's Theorem for the function f(x) =log$\left[\frac{x^2+ab}{x(a+b)}\right]$ in $[a,b]$ (x$\neq 0$)	[L2][CO2]	**[6M]**
	b) Verify Lagrange's mean value theorem for f(x)=$x^3 - x^2 - 5x + 3$ in $[0,4]$.	[L2][CO2]	**[6M]**
3	a) Verify Lagrange's Mean value theorem for the functions $f(x) = x(x-1)(x-2) in \left[0,\frac{1}{2}\right]$.	[L2][CO2]	**[6M]**
	b) Expand $\log_e x$ in powers of (x-1) and hence evaluate $\log 1.1$ correct to 4 decimal places using Taylor's theorem.	[L2][CO2]	**[6M]**
4	a) Express the polynomial $2x^3 + 7x^2 + x$-6 in power of $(x-2)$ assigning Taylor's series.	[L3][CO2]	**[6M]**
	b) Using Maclaurin's series expand $\tan x$ up to the fifth power of x and hence find the series for log (sec x).	[L3][CO2]	**[6M]**
5	a) Expand $\sin x$ powers of (x- $\frac{\pi}{2}$) up to the term containing $(x - \frac{\pi}{2})^4$ assigning Taylor's series.	[L3][CO2]	**[6M]**
	b) If $u = \frac{x+y}{1-xy}$ $and\ v = tan^{-1}x + tan^{-1}y\ ,find\ \frac{\partial(u,v)}{\partial(x,y)}$?	[L1][CO2]	**[6M]**
6	a) If $u = f(y-z, z-x, x-y)$ $prove\ that\ \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ by using Chain rule .	[L3][CO2]	**[6M]**
	b) $u = sin^{-1}(x-y), where\ x = 3t, y = 4t^3\ , then\ show\ that\ \frac{du}{dt} = \frac{3}{\sqrt{1-t^2}}\ by\ total\ derivative.$	[L2][CO2]	**[6M]**
7	a) If $u = x^2 - 2y;\ v = x + y + z,\ w = x - 2y + 3z$,then find Jacobian $J\left(\frac{u,v,w}{x,y,z}\right)$.	[L1][CO2]	**[6M]**
	b) Verify if $u = 2x - y + 3z$,$v = 2x - y - z,\ w = 2x - y + z$ are functionally dependent and if so, find the relation between them.	[L2][CO2]	**[6M]**
8	a) Examine the function for extreme values $f(x,y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$; (x>0,y>0).	[L4][CO2]	**[6M]**
	b) Find the minimum value of $x^2 + y^2 + z^2\ given\ x + y + z = 3a$.	[L1][CO2]	**[6M]**
9	a) Find the stationary points of $u(x,y) = sinx.siny.sin(x+y)\ where\ 0 < x < \pi, 0 < y < \pi$ and find the maximum of u.	[L1][CO2]	**[6M]**
	b) Find the shortest distance from origin to the surface $xyz^2 = 2$.	[L1][CO2]	**[6M]**
10	a) Find a point on the plane $3x + 2y + z - 12 = 0,$ which is nearest to the origin.	[L1][CO2]	**[6M]**
	b) Find shortest and longest distance from the point $(3,1,-1) to\ the\ sphere\ x^2 + y^2 + z^2 = 4$	[L1][CO2]	**[6M]**

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UNIT –III
INTEGRAL CALCULUS

1	a) Evaluate $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$	[L5][CO3]	**[6M]**
	b) Evaluate $\int_0^{\pi} \theta \sin^8\theta \cos^4\theta d\theta$	[L5][CO3]	**[6M]**
2	a) Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$	[L5][CO3]	**[6M]**
	b) Evaluate the following improper integrals i) $\int_1^{\infty} \frac{1}{x^4}dx$. ii) $\int_0^1 \frac{1}{\sqrt{x}}dx$.	[L5][CO3]	**[6M]**
3	a) Show that $\int_{-\infty}^{\infty} \frac{1}{1+x^2}dx = \pi$.	[L1][CO3]	**[6M]**
	b) Evaluate $\int_0^5 \int_0^{x^2} x(x^2+y^2)dxdy$	[L5][CO3]	**[6M]**
4	a) Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2+y^2)dydx$	[L5][CO3]	**[6M]**
	b) Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2+y^2)dxdy$.	[L5][CO3]	**[6M]**
5	a) Evaluate $\int_0^4 \int_0^{x^2} e^{\frac{y}{x}}dydx$	[L5][CO3]	**[6M]**
	b) Evaluate the integral by changing the order of integration $\int_0^{\infty}\int_x^{\infty} \frac{e^{-y}}{y}dydx$.	[L3][CO3]	**[6M]**
6	Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} (xy)dydx$ and hence evaluate the same.	[L3][CO3]	**[12M]**
7	a) By changing order of integration, evaluate $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx$.	[L3][CO3]	**[6M]**
	b) Evaluate the integral by transforming into polar coordinates $\int_0^a \int_0^{\sqrt{a^2-x^2}} y\sqrt{x^2+y^2}\,dxdy$.	[L3][CO3]	**[6M]**
8	a) Evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}dxdy$ by converting to polar coordinates.	[L3][CO3]	**[6M]**
	b) Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} (x^2+y^2)dxdy$ by changing into polar coordinates	[L3][CO3]	**[6M]**
9	a) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$.	[L5][CO3]	**[6M]**
	b) Evaluate $\int_0^{log2} \int_0^x \int_0^{x+y} e^{x+y+z} dzdydx$.	[L5][CO3]	**[6M]**
10	a) Evaluate $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x+y+z)dxdydz$	[L5][CO3]	**[6M]**
	b) Evaluate $\int_1^e \int_1^{logy} \int_1^{e^x} logz\, dzdxdy$.	[L5][CO3]	**[6M]**

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UNIT –IV
VECTOR DIFFERENTIATION

1	a) Find $grad\ f$ if $f = xz^4 - x^2y$ at a point $(1,-2,1)$ .Also find $\lvert\nabla f\rvert$	[L1][CO4]	**[6M]**
	b) If $\bar{r} = x\vec{i} + y\vec{j} + z\vec{k}$ then show that $\nabla r = \frac{\bar{r}}{r}$	[L2][CO4]	**[6M]**
2	a) Find the directional derivative of $2xy + z^2$ at $(1,-1,3)$ in the direction of $\vec{i} + 2\vec{j} + 3\vec{k}$	[L1][CO4]	**[6M]**
	b) Find the directional derivative of $xyz^2 + xz$ at $(1,1,1)$ in the direction of normal to the surface $3xy^2 + y = z$ at $(0,1,1)$	[L1][CO4]	**[6M]**
3	a) Find the angle between the normal to the surface $xy = z^2$ at the points $(4,1,2)$ *and* $(3,3,-3)$. .	[L1][CO4]	**[6M]**
	b) Find the maximum or greatest value of the directional derivative of $f = x^2yz^3$ at the point $(2,1,-1)$.	[L1][CO4]	**[6M]**
4	a) Find the divergence of $\bar{f} = (xyz)\vec{i} + (3x^2y)\vec{j} + (xz^2 - y^2z)\vec{k}$.	[L1][CO4]	**[6M]**
	b) Show that $\bar{f} = (x + 3y)\vec{i} + (y - 2z)\vec{j} + (x - 2z)\vec{k}$ is *solenoidal.*	[L2][CO4]	**[6M]**
5	a) Find $div\bar{f}$ *if* $\bar{f} = grad(x^3 + y^3 + z^3 - 3xyz)$.	[L1][CO4]	**[6M]**
	b) Find $curl$ of the vector $\bar{f} = (x + y + 1)\vec{i} + \vec{j} - (x + y)\vec{k}$.	[L1][CO4]	**[6M]**
6	a) Prove that $\bar{f} = (y + z)\vec{i} + (z + x)\vec{j} + (x + y)\vec{k}$ is *irrotational.*	[L6][CO4]	**[6M]**
	b) Find $curl\ \bar{f}$ if $\bar{f} = grad(x^3 + y^3 + z^3 - 3xyz)$.	[L1][CO4]	**[6M]**
7	a) Find 'a' if $\bar{f} = y(ax^2 + z)\vec{i} + x(y^2 - z^2)\vec{j} + 2xy(z - xy)\vec{k}$ is *solenoidal*.	[L1][CO4]	**[6M]**
	b) If $\bar{f} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational then find the constants *a,b and c*.	[L1][CO4]	**[6M]**
8	a) Find $\nabla \times (\nabla \times \bar{f}),\ if\ \bar{f} = (x^2y)\vec{i} - (2xz)\vec{j} + (2yz)\vec{k}$.	[L1][CO4]	**[6M]**
	b) Prove that $div(curl\bar{f}) = 0$.	[L1][CO4]	**[6M]**
9	a) Show that that $\nabla(r^n) = n\, r^{n-2}\bar{r}$	[L2][CO4]	**[6M]**
	b) Prove that $curl(\emptyset\bar{f}) = (grad\emptyset) \times \bar{f} + \emptyset(curl\bar{f})$	[L6][CO4]	**[6M]**
10	a) Prove that $\nabla.(\bar{f} \times \bar{g}) = \bar{g}.(\nabla \times \bar{f}) - \bar{f}.(\nabla \times \bar{g})$	[L6][CO4]	**[6M]**
	b) Show that $\nabla \times (\bar{f} \times \bar{g}) = \bar{f}(\nabla.\bar{g}) - \bar{g}(\nabla.\bar{f}) + (\bar{g}.\nabla)\bar{f} - (\bar{f}.\nabla)\bar{g}$	[L2][CO4]	**[6M]**

Course Code: 20HS0380 **R20**

UNIT –V

VECTOR INTEGRATION & INTEGRAL THEOREMS

1	a) If $\bar{F} = (5xy - 6x^2)\vec{i} + (2y - 4x)\vec{j}$ .Evaluate $\int_c \bar{F}.d\bar{r}$ along the curve 'c' in xy-plane $y = x^3$ from $(1,1) to (2,8)$.	[L5][CO5]	**[6M]**
	b) Find the work done by a force $\bar{F} = (2y+3)\vec{i} + (xz)\vec{j} + (yz - x)\vec{k}$ when it moves a particle from $(0,0,0) to (2,1,1)$ along the curve $x = 2t^2; y = t ; z = t^3$.	[L1][CO5]	**[6M]**
2	If $\bar{F} = (x^2 + y^2)\vec{i} - (2xy)\vec{j}$. Evaluate $\int_c \bar{F}.d\bar{r}$ where 'c' is the rectangle in xy-plane bounded by $y = 0; y = b \ and\ x = 0; x = a.$	[L5][CO5]	**[12M]**
3	a) Evaluate $\int_s \bar{F}.\bar{n}ds.$ where $\bar{F} = 18z\vec{i} - 12\vec{j} + 3y\vec{k}$ and 's' is the part of the surface of the plane $2x + 3y + 6z = 12$ located in the first octant.	[L5][CO5]	**[6M]**
	b) Evaluate $\int_s \bar{F}.\bar{n}ds.$ where $\bar{F} = 12x^2y\vec{i} - 3yz\vec{j} + 2z\vec{k}$ and 's' is the portion of the plane $x + y + z = 1$ located in the first octant.	[L5][CO5]	**[6M]**
4	a) If $\bar{F} = 2xz\vec{i} - x\vec{j} + y^2\vec{k}$. Evaluate $\int_v \bar{F}.dv$ where 'v' is the region bounded by the surfaces $x = 0; x = 2 : y = 0; y = 6\ and\ z = x^2; z = 4$.	[L5][CO5]	**[6M]**
	b) If $\bar{F} = (2x^2 - 3z)\vec{i} - 2xy\vec{j} - 4x\vec{k}$ then Evaluate $\int_v \nabla.\bar{F}\ dv$ where 'v' is the closed region bounded by $x = 0; y = 0; z = 0\ and\ 2x + 2y + z = 4$.	[L5][CO5]	**[6M]**
5	a) State Gauss's divergence theorem.	[L1][CO5]	**[2M]**
	b) By transforming into triple integral, Evaluate $\iint_s x^3dydz + x^2ydzdx + x^2zdxdy$ where 's' is the closed surface consisting of the cylinder $x^2 + y^2 = a^2$ and the circular discs $z = 0; z = b$.	[L5][CO5]	**[10M]**
6	Verify Gauss's divergence theorem for $\bar{F} = (x^3 - yz)\vec{i} - 2x^2y\vec{j} + z\vec{k}$ taken over the surface of the cube bounded by the planes $x = y = z = a$ and coordinate planes.	[L2][CO5]	**[12M]**
7	a) Apply Green's theorem to Evaluate $\oint_c (2x^2 - y^2)dx + (x^2 + y^2)dy$ where 'c' is the enclosed by the x-axis and upper half of the circle $x^2 + y^2 = a^2$.	[L1][CO5]	**[6M]**
	b) Evaluate by Green's theorem $\oint_c (y - sinx)dx + cosxdy$ where 'c' is the triangle enclosed by the lines $y = 0, x = \frac{\pi}{2}\ and\ \pi y = 2x.$	[L5][CO5]	**[6M]**
8	a) State Green's theorem in a plane.	[L1][CO5]	**[2M]**
	b) Verify Green's theorem in a plane for $\oint_C (x^2 - xy^3)dx + (y^2 - 2xy)dy$ where 'c' is a square with vertices $(0,0)(2,0)(2,2)\ and\ (0,2)$.	[L2][CO5]	**[10M]**
9	Verify Stoke's theorem for $\bar{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ taken round the rectangle bounded by the lines $x = \pm a$, $y = \pm b$.	[L2][CO5]	**[12M]**
10	a) State Stoke's theorem.	[L1][CO5]	**[2M]**
	b) Verify Stoke's theorem for $\bar{F} = x^2\vec{i} + xy\vec{j}$ integrated round the square in the plane $z = 0$,whose sides are along the line $x = 0, y = 0 ; x = a, y = a.$	[L2][CO5]	**[10M]**

Prepared by: Dept. of Mathematics